Activity: bisecting the multiplier

Chapter 4, Multi-Item and Constrained Systems

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Work in pairs. 20 minutes.

Names: _____

The items. The distributor that stocks the mower deck belt orders three of its items from three suppliers. Each order costs $k = \$40$, the carrying charge is $i = 25\%$ a year, demand is constant and known, and no shortages are allowed.

Item		λ_j, a year	c_j
1	Mower deck belt	3,000	\$12.00
2	Blade kit	2,400	\$5.00
3	Spindle kit	600	\$40.00

The controller limits the average investment in these three items to **\$3,000**. Under a budget the constrained quantities are

$$Q_j(\theta) = \sqrt{\frac{2k\lambda_j}{(i+\theta)c_j}}, \qquad \text{investment} = \sum_j c_j \frac{Q_j(\theta)}{2}$$

Report quantities to two decimals and dollars to the cent.

Part A. Predict

The budget is about three quarters of what the unconstrained quantities need. Which item's quantity will be cut hardest, in proportion to its unconstrained value?

Part B. Does the limit bind?

Compute each unconstrained Q_j^* and the investment it requires. Does the \$3,000 limit bind?

Part C. Four bisection steps

Bracket the multiplier with $\theta = 0$ and $\theta = 1$. At each step compute the three quantities at the midpoint, then the investment, and keep the half in which the investment crosses \$3,000.

Step	Low	High	Midpoint θ	Q_1	Q_2	Q_3	Investment	Keep
1								
2								
3								
4								

After four steps the multiplier is bracketed. Twenty more steps give $\theta^* = 0.20844$. At θ^*, compute the three quantities, the annual ordering and holding costs, and the cost of the limit against the unconstrained answer. Check that the holding cost is what it has to be.

Part D. What the multiplier says

1. The controller asks what one more dollar of budget would be worth. Answer in dollars a year, and say why the answer holds only for a small change.
2. Compute Q_j/Q_j^* for each item. Compare with your prediction.

Part E, if you have time. No bisection needed

Use your answer to Part D.2 to find θ^* directly from the budget, without bisecting. Why would the same shortcut fail if the limit were on floor space?

Exit ticket (tear off and hand in)

Name: _____

1. In one sentence, what does $\theta^* = 0.208$ tell the controller?

2. True or false, and why: at the constrained quantities the annual ordering and holding costs are equal.