Activity: evaluating a policy from the Poisson table

Chapter 8, Continuous and Periodic Review Systems

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Work in pairs. 25 minutes.

Names: _____

The item. The storeroom stocks a capacitor bank controller. Crews draw **24** a year, one at a time, as a Poisson process. The supplier quotes a constant lead time of **two months**. A controller costs **$1,800**, the carrying charge is **25%** a year, and unmet demand is backordered. The storeroom runs $(r, Q) = (5, 3)$.

Notation. θ is the mean lead time demand. $G(x) = P\{D(L) \le x\}$ is the Poisson distribution function and $G^1(x) = E[(D(L) - x)^+]$ its first order loss function. For a position j, the ready rate is $G(j-1)$ and the expected backorders are $G^1(j)$. Then

$$\overline{RR} = \frac{1}{Q}\sum_{j=r+1}^{r+Q} G(j-1), \qquad \bar{B} = \frac{1}{Q}\sum_{j=r+1}^{r+Q} G^1(j), \qquad \bar{I} = \frac{Q+1}{2} + r - \theta + \bar{B}$$

The table. Poisson lead time demand with mean 4.

x	0	1	2	3	4	5	6	7	8	9	10	11	12
$G(x)$	0.0183	0.0916	0.2381	0.4335	0.6288	0.7851	0.8893	0.9489	0.9786	0.9919	0.9972	0.9991	0.9997
$G^1(x)$	4.0000	3.0183	2.1099	1.3480	0.7815	0.4103	0.1954	0.0848	0.0336	0.0123	0.0041	0.0013	0.0004

Report probabilities and averages to four decimal places.

Part A. Predict

A colleague says: “We reorder at 5, so we run out whenever lead time demand exceeds 5. The fill rate is $G(5) = 0.7851$.” Is that too high, too low, or right? Why?

Part B. Evaluate $(5, 3)$

1. Compute θ, showing the unit conversion. Which values can the inventory position take, and with what probability each?
2. For each position, read the ready rate and the expected backorders from the table. Average them to get $\overline{RR}$ and $\bar{B}$. Why is the fill rate equal to the ready rate here?
3. Compute $\bar{I}$.
4. Check the ready rate by a second route: $\overline{RR} = 1 - \left[G^1(r) - G^1(r+Q)\right]/Q$.

Part C. Meet a 95% fill rate

1. Holding $Q = 3$, find the smallest r whose fill rate is at least 0.95. Show each r you tried.
2. Report $\bar{B}$ and $\bar{I}$ at that r.
3. Compute h. How much more stock is on the shelf, on average, than at $r = 5$, and what does that cost a year?

Part D, if you have time. What the planning package does

The storeroom's planning package sets $r = \theta + z\sigma$, with $\sigma = \sqrt{\theta}$ and $z = 1.645$ "for 95% service", and rounds up.

1. What r does it set? What fill rate does that policy actually deliver, at $Q = 3$?
2. What does the extra stock cost a year against your answer to Part C?
3. Read the $G(x)$ row for the smallest x with $G(x) \geq 0.95$. What does that tell you about whether the normal or the meaning of "95%" caused the gap?

Exit ticket (tear off and hand in)

Name: _____

1. In one sentence: why is the fill rate of an (r, Q) policy higher than $G(r)$?

2. True or false, and why: holding r fixed, raising Q lowers the fill rate.