# Activity: lead time demand by conditioning

Chapter 8, Continuous Review Systems

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Work in pairs. 20 minutes.

Names: _____

**The item.** The utility storeroom stocks the pad-mount transformer. Crews draw **45** a year, one at a time, as a Poisson process, so demand in a month has mean and variance $45/12 = 3.75$. The supplier's lead time is no longer constant. From its record of past deliveries:

| Lead time $L$, months | 1 | 2 | 3 |
|---|---|---|---|
| $P\{L = j\}$ | 0.3 | 0.5 | 0.2 |

**Notation.** $D(L)$ is the demand over a lead time. Given $L = j$ months, the lead time is fixed, so $D(L)$ is Poisson with mean and variance $3.75j$. The law of total expectation averages a conditional quantity over the lead time:

$$E[X] = \sum_j E[X \mid L = j]\, P\{L = j\}$$

Report to four decimal places.

## Part A. Predict

A planner reasons: "Demand is Poisson, so lead time demand is Poisson with mean $3.75 \times E[L]$, and its variance equals its mean." Is that variance too high, too low, or right? Why?

## Part B. Condition on the lead time

For each lead time, fill the conditional mean, the conditional variance and the conditional second moment $E[D(L)^2 \mid L = j]$.

| $j$ | $P\{L = j\}$ | $E[D(L) \mid L = j]$ | $Var[D(L) \mid L = j]$ | $E[D(L)^2 \mid L = j]$ |
|---|---|---|---|---|
| 1 | 0.3 | | | |
| 2 | 0.5 | | | |
| 3 | 0.2 | | | |

## Part C. Average over the lead time

Write each sum out term by term before you add it.

1. The mean, $E[D(L)] = \sum_j E[D(L) \mid L = j]\, P\{L = j\}$.

2. The second moment the same way, and then $Var[D(L)] = E[D(L)^2] - \left(E[D(L)]\right)^2$.

3. Split the variance by the law of total variance, $Var[D(L)] = E\big[\,Var[D(L) \mid L]\big] + Var\big[E[D(L) \mid L]\big]$. Compute each term from your table, and check that they add to your answer to 2.

## Part D. Check against the three cases

1. Compute $E[L]$ and $Var[L]$, in months.
2. Case 3 of the chapter's table of lead time demand cases gives $Var[D(L)] = E[L]\,Var[D] + Var[L](E[D])^2$, with $D$ the demand in one month. Does it agree with Part C?
3. Which term is the customers and which the supplier? What share of the variance does the supplier contribute?

## Part E, if you have time. A guaranteed lead time

For a fee, the supplier will guarantee two months on every order. Compute the mean and variance of lead time demand under the guarantee, and the variance to mean ratio before and after. Which family would you fit in each case?

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**Exit ticket** (tear off and hand in)

Name: _____

1. In one sentence: why is the variance of lead time demand larger than its mean when demand is Poisson?

2. True or false, and why: when demand is Poisson, $Var[D(L)] = E[L]\,Var[D]$.